Geometric Series

1.

(a) A geometric series has first term a and common ratio r. Prove that the sum of the first n terms is given by

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r}.$$ [3]

(b) The fourth term of a geometric series is -108 and the seventh term is 4.

(i) Find the common ratio of the series.

(ii) Find the sum to infinity of the series. [6]

2.

(a) The eighth and ninth terms of a geometric series are 576 and 2304 respectively. Find the fifth term of the geometric series. [3]

(b) Another geometric series has first term a and common ratio r. The third term of this geometric series is 24. The sum of the second, third and fourth terms of the series is -56.

(i) Show that r satisfies the equation

$$3r^2 + 10r + 3 = 0.$$

(ii) Given that $|r| < 1$, find the value of r and the sum to infinity of the series. [8]

3.

(a) A geometric series has first term a and common ratio r. Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{a\left(1-r^n\right)}{1-r} .$$ [3]

(b) The sum of the first five terms of a geometric series is 275. The sum to infinity of the series is 243. Find the common ratio and the first term of the geometric series. [6]

4.

A geometric series has first term a and common ratio r. The sum of the second and third terms of the series is -216. The sum of the fifth and sixth terms of the series is 8.

(a) Prove that $r = -\frac{1}{3}$. [5]

(b) Find the sum to infinity of the series. [3]

5.

A rich businessman makes one donation per year to a certain charity. He starts by donating £100 in the first year. In each subsequent year, the value of the donation is 1·2 times the value of the previous year's donation.

(a) Find the value of the businessman's donation in the 12th year. Give your answer correct to the nearest pound. [2]

(b) After receiving the nth donation, the charity's treasurer calculates that over the years, the businessman has donated a **total** of £15 474, correct to the nearest pound. Find the value of n. [5]

Marking Scheme

1.

(a)		$S_n = a + ar + \ldots + ar^{n-1}$	(at least 3 terms, one at each end)	B1
		$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$		
		$S_n - rS_n = a - ar^n$	(multiply first line by r and subtract)	M1
		$(1 - r)S_n = a(1 - r^n)$		
		$S_n = \dfrac{a(1 - r^n)}{1 - r}$	(convincing)	A1
(b)	(i)	$ar^3 = -108$ and $ar^6 = 4$		B1
		$r^3 = \dfrac{4}{-108}$	(o.e.)	M1
		$r = -{}^1/_3$	(c.a.o.)	A1
	(ii)	$a \times (-{}^1/_3)^3 = -108 \Rightarrow a = 2916$		
			(f.t. candidate's derived value for r)	B1
		$S_\infty = \dfrac{2916}{1 - (-{}^1/_3)}$	(use of formula for sum to infinity)	
			(f.t. candidate's derived values for r and a)	M1
		$S_\infty = 2187$	(c.a.o.)	A1

2.

(a)		$r = \dfrac{2304}{576} = 4$	(c.a.o.)	B1
		$t_5 = \dfrac{576}{4^3}$	(f.t. candidate's value for r)	M1
		$t_5 = 9$	(c.a.o.)	A1
(b)	(i)	$ar^2 = 24$		B1
		$ar + ar^2 + ar^3 = -56$		B1
		An attempt to solve the candidate's equations simultaneously by eliminating a		M1
		$\dfrac{r^2}{r + r^2 + r^3} = -\dfrac{24}{56} \Rightarrow 3r^2 + 10r + 3 = 0$	(convincing)	A1
	(ii)	$r = -\dfrac{1}{3}$	($r = -3$ discarded, c.a.o.)	B1
		$a = 216$		
			(f.t. candidate's derived value for r, provided $\lvert r \rvert < 1$)	B1
		$S_\infty = \dfrac{216}{1 - (-{}^1/_3)}$	(use of formula for sum to infinity)	
			(f.t. candidate's derived values for r and a)	M1
		$S_\infty = 162$	(f.t. candidate's derived values for r and a)	A1

3.

(*a*)	$S_n = a + ar + \ldots + ar^{n-1}$	(at least 3 terms, one at each end)	B1
	$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$		
	$S_n - rS_n = a - ar^n$	(multiply first line by r and subtract)	M1
	$(1 - r)S_n = a(1 - r^n)$		
	$S_n = \dfrac{a(1 - r^n)}{1 - r}$	(convincing)	A1
(*b*)	**Either:** $\dfrac{a(1 - r^5)}{1 - r} = 275$		
	Or: $a + ar + ar^2 + ar^3 + ar^4 = 275$		B1
	$\dfrac{a}{1 - r} = 243$		B1
	An attempt to solve these equations simultaneously by eliminating a		M1
	$243r^5 = -32$ (or $-243r^5 = 32$)		A1
	$r = -\dfrac{2}{3}$	(c.a.o.)	A1
	$a = 405$	(f.t. candidate's derived value for r)	A1

4.

(*a*)	$ar + ar^2 = -216$		B1
	$ar^4 + ar^5 = 8$		B1
	A correct method for solving the candidate's equations simultaneously		
	e,g multiplying the first equation by r^3 and subtracting		
	or eliminating a and $(1 + r)$		M1
	$-216r^3 = 8$	(o.e.)	A1
	$r = -{}^1/_3$	(convincing)	A1
(*b*)	$a \times (-{}^1/_3) \times (1 - {}^1/_3) = -216 \Rightarrow a = 972$		B1
	$S_\infty = \dfrac{972}{1 - (-{}^1/_3)}$	(correct use of formula for S_∞, f.t. candidate's derived value for a)	M1
	$S_\infty = 729$	(f.t. candidate's derived value for a)	A1

5.

(*a*)	$a = 100$, $r = 1{\cdot}2$		
	Value of donation in 12th year = $100 \times 1{\cdot}2^{11}$		M1
	Value of donation in 12th year = £743		A1
(*b*)	$100 \times \dfrac{(1 - 1{\cdot}2^n)}{1 - 1{\cdot}2} = 15474$		M1
	$1 - 1{\cdot}2^n = 154{\cdot}74 \times (-0{\cdot}2)$		m1
	$1{\cdot}2^n = 31{\cdot}948$		A1
	$n = \dfrac{\log 31{\cdot}948}{\log 1{\cdot}2}$		m1
	$n = 19$	cao	A1